MATHS OLD LC P1

Section A	Concepts and Skills	150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

A shopkeeper bought 25 school blazers at €30 each and 25 trousers at €20 each.

(a) Find the total cost to the shopkeeper.

Suggested Solution

25×30 = 750
25×20 = 500
Total = €1250 ANS

(b) The shopkeeper sells a blazer and a trousers as a set for €89·95. Find her profit on this transaction.

(i) 30+20 = 50 Cost

(ii) 89.95 − 50.00 = 39.95 profit

€39.95 ANS

(c) The shopkeeper sells 22 blazer and trouser sets at €89·95 each. She sells the remaining 3 sets at a discount of 20% on the selling price. Find her mark up (profit as a percentage of cost price) on the total transaction.

(i) 22×89.95 = 1978.9 sells

(ii) $\frac{89.95}{100} \times \frac{20}{1}$ = 17.99 discount ∴ sells 3 at (89.95 − 17.99) each. 71.96 Euro.

∴ 3×71.96 = 215.88 for 3

(iii) Total Selling
1978.9 + 215.88.
= 2194.78

(iv) profit = 2194.78 − 1250
= 944.78 ∴ % Profit
= $\frac{944.78}{1250} \times \frac{100}{1}$ = 75.58%

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Question 2 **(25 marks)**

Let $z_1 = 5 - i$ and $z_2 = 4 + 3i$, where $i^2 = -1$.

(a) **(i)** Find $z_1 - z_2$.

$5 - i - (4 + 3i)$

$5 - i - 4 - 3i$

$5 - 4 - i - 3i \quad = \underline{1 - 4i}$

(ii) Verify that $|z_1 - z_2| = |z_2 - z_1|$.

$\underline{z_2 - z_1} = 4 + 3i - (5 - i)$

$4 + 3i - 5 + i$

$\underline{-1 + 4i}$

$|1 - 4i| = |-1 + 4i|$

$\sqrt{1^2 + (-4)^2} = \sqrt{(-1)^2 + (4)^2}$

$\sqrt{1 + 16} = \sqrt{1 + 16} \Rightarrow \underline{\sqrt{17} = \sqrt{17}}$ $\underline{\text{True}}$

(iii) Give a reason why $|z - w| = |w - z|$ will always be true, for any complex numbers z and w.

$|z - w|$ means the Distance between the complex numbers z and w

and so does $|w - z|$ – mean distance between w and z

$\therefore$ Same Thing

(b) Find a complex number z_3 such that $z_1 = \dfrac{z_2}{z_3}$.

Give your answer in the form $a + bi$, where $a, b \in \mathbb{R}$.

(i) $z_1 = \dfrac{z_2}{z_3}$ sub in z_1 and z_2.

$\dfrac{5 - i}{1} = \dfrac{4 + 3i}{z_3} \Rightarrow z_3(5 - i) = 4 + 3i$

$z_3 = \dfrac{4 + 3i}{5 - i}$ $\underline{\text{A Durow}}$

(ii) $z_3 = \dfrac{4 + 3i}{5 - i} \times \dfrac{5 + i}{5 + i} = \dfrac{4(5 + i) + 3i(5 + i)}{5(5 + i) - i(5 + i)} = \dfrac{20 + 4i + 15i + 3i^2}{25 + 5i - 5i - i^2}$

$\dfrac{20 + 19i + 3(-1)}{25 - i^2} = \dfrac{20 + 19i - 3}{25 - (-1)} = \dfrac{20 - 3 + 19i}{25 + 1} = \dfrac{17 + 19i}{26} = \dfrac{17}{26} + \dfrac{19}{26}i$

$\underline{a + bi}$

Question 3 **(25 marks)**

(a) **(i)** Solve for x:

$$2(4-3x)+12=7x-5(2x-7).$$

$8-6x+12=7x-10x+35$

$20-6x=-3x+35$

$20-35=-3x+6x$

$3x=-15$

$x=-5$

(ii) Verify your answer to **(i)** above.

Sub in $x=-5$

$2(4-3(-5))+12=7(-5)-5(2(-5)-7)$

$2(4+15)+12=-35-5(-10-7)$

$2(19)+12=-35-5(-17) \Rightarrow 38+12=-35+85$

$50=50$ True.

(b) Solve the simultaneous equations:

$$x+y=7$$

$$x^2+y^2=25.$$

(I) $x=7-y$

(II) $x^2+y^2=25$

$(7-y)^2+y^2=25$

$49+y^2-14y+y^2-25=0$

$2y^2-14y+24=0$

$y^2-7y+12=0$

$y \quad -4 \qquad y-4=0$

$y \quad -3 \qquad y=4$

$y-3=0$

$y=3$

(III)

$y=4$	$y=3$
$x=7-y$	$x=7-3$
$x=7-4$	$x=4$
$x=3$	
(3, 4)	(4, 3)
	Answers

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Question 4 **(25 marks)**

(a) Solve the equation $x^2 - x - 6 = 0$.

(1) Could use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 1$, $b = -1$, $c = -6$

(2) Factors

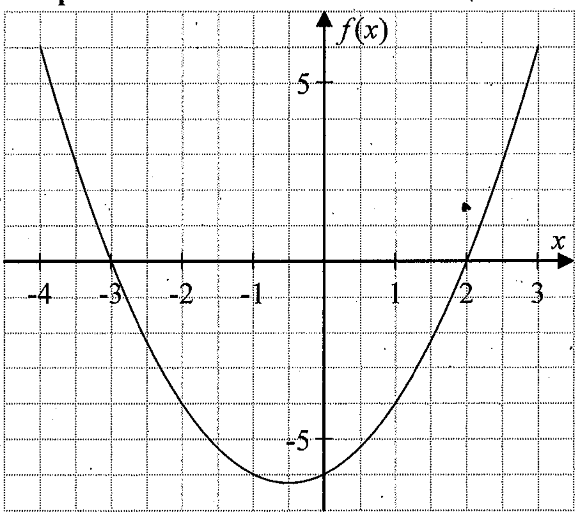
$x - 3$

$x + 2$ $x - 3 = 0$ | $x + 2 = 0$

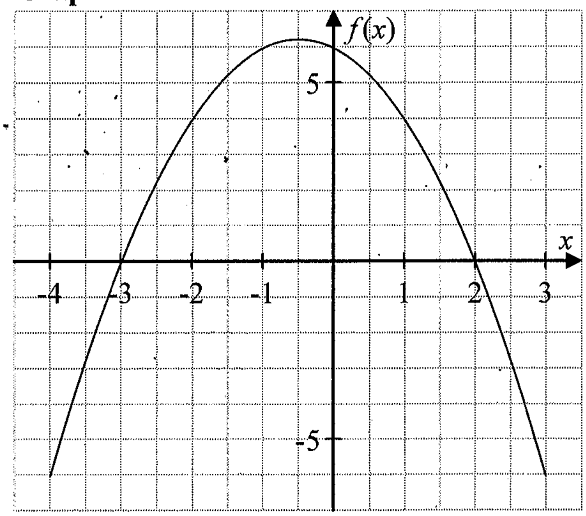
$x = 3$ | $x = -2$

(b) The graphs of four quadratic functions are shown below.

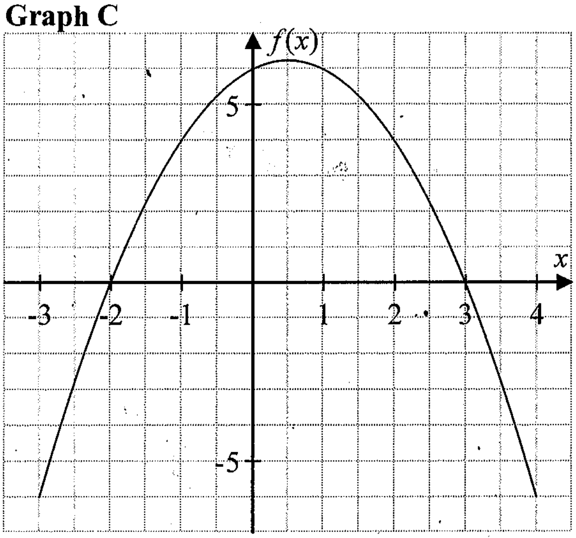
Graph A



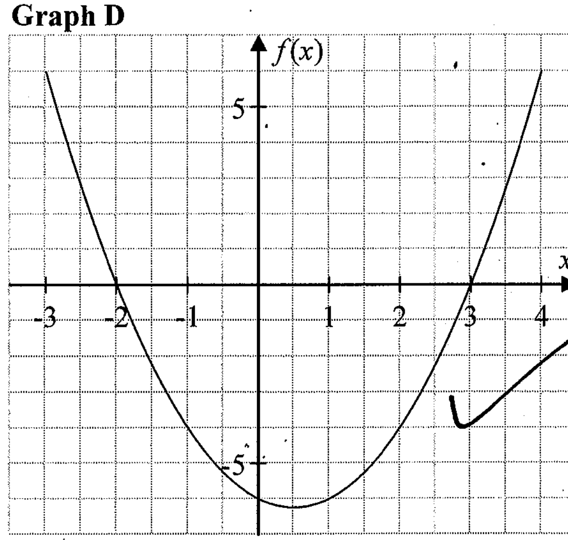
Graph B



Graph C



Graph D



Which of the graphs above is that of the function $f : x \mapsto x^2 - x - 6$, where $x \in \mathbb{R}$?

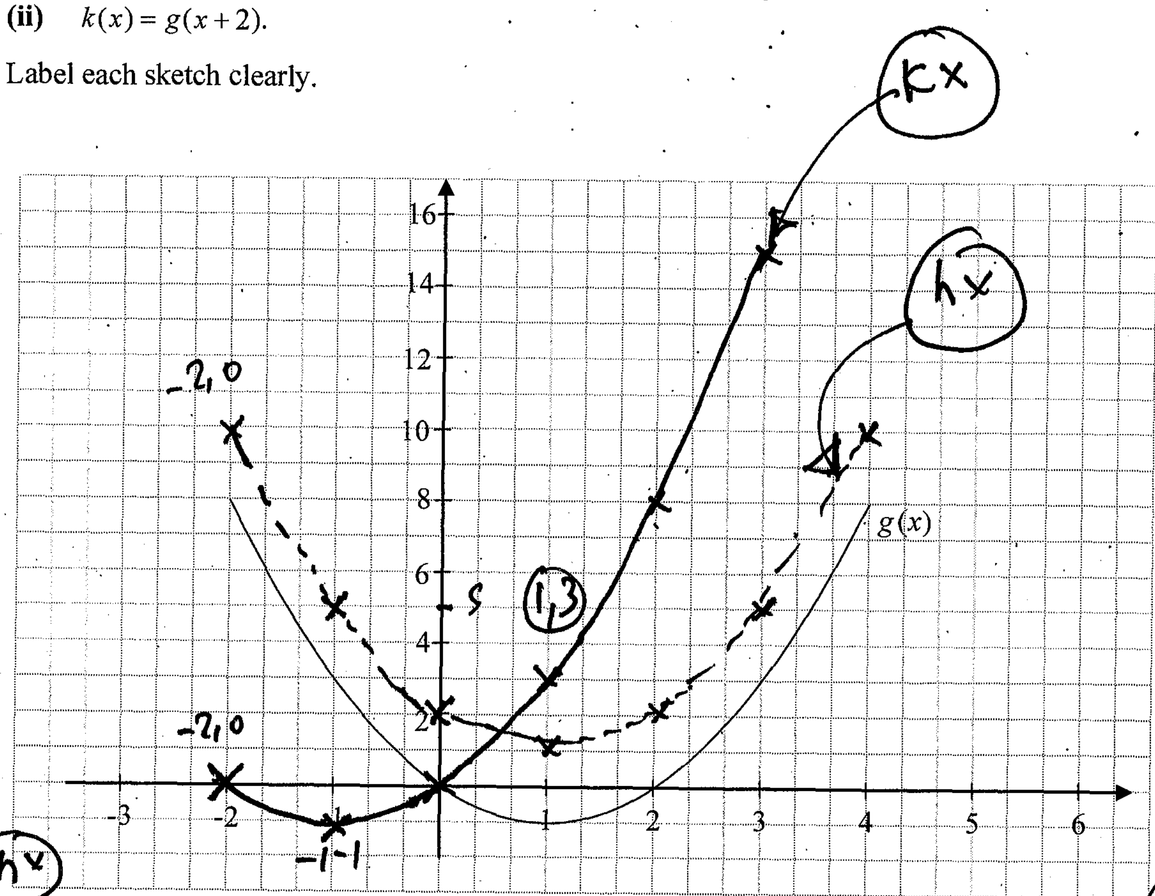
graph D.

(c) The graph of $g(x) = x^2 - 2x$, where $x \in \mathbb{R}$, is shown on the diagram below.
On the same diagram, sketch the graph of each of the functions:

(i) $h(x) = g(x) + 2$

(ii) $k(x) = g(x + 2)$.

Label each sketch clearly.



(i) $h(x) = g(x) + 2 = x^2 - 2x + 2$

replace x with x+2 in g(x)

(ii) $k(x) = g(x+2) = (x+2)^2 - 2(x+2)$

$x^2 + 4 + 4x - 2x - 4 = x^2 + 2x$

graphs - hx

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$-2x$	4	2	0	-2	-4	-6	-8
$+2$	2	2	2	2	2	2	2
y	10	5	2	1	2	5	10

x	-2	-1	0	1	2	3	4
y	10	5	2	1	2	5	10

kx

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$+2x$	-4	-2	0	2	4	6	8
y	0	-1	0	3	8	15	24

x	-2	-1	0	1	2	3	4
y	0	-1	0	3	8	15	24

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Question 5 **(25 marks)**

The function f is defined as $f: x \mapsto x^3 + 3x^2 - 9x + 5$, where $x \in \mathbb{R}$.

(a) **(i)** Find the co-ordinates of the point where the graph of f cuts the y-axis.

Cuts y axis at $x = 0$

$y = 0^3 + 3(0)^2 - 9(0) + 5 = 5$

$\therefore (0, 5)$

(ii) Verify that the graph of f cuts the x-axis at $x = -5$.

sub in $x = -5$ and if $y = 0$ it does

$(-5)^3 + 3(-5)^2 - 9(-5) + 5$

$-125 + 75 + 45 + 5 = -5$ $\therefore$ true

(b) Find the co-ordinates of the local maximum turning point and of the local minimum turning point of f.

(i) $f(x) = x^3 + 3x^2 - 9x + 5$

$f'(x) = 3x^2 + 6x - 9$

$0 = 3x^2 + 6x - 9$

$\frac{\cdot}{\cdot} 3$

$0 = x^2 + 2x - 3$

or use -b formula

(ii) factorize solve

$x + 3$

$x - 1$

$x + 3 = 0$ | $x - 1 = 0$

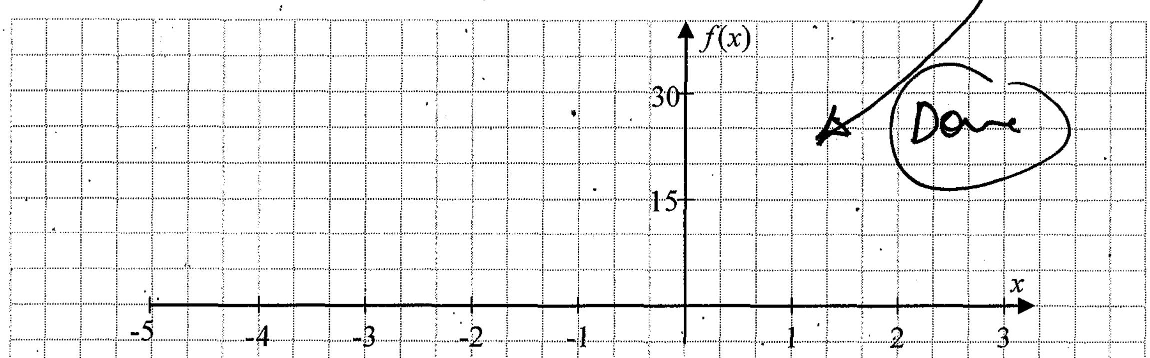
$x = -3$ | $x = 1$

(iii) $x = 1$	$x = -3$
$y = 1^3 + 3(1)^2 - 9(1) + 5$	$y = (-3)^3 + 3(-3)^2 - 9(-3) + 5$
$y = 1 + 3 - 9 + 5$	$y = -27 + 27 + 27 + 5$
$y = 0$	$y = 32$
$\therefore 1, 0$	$\therefore (-3, 32)$

$(-3, 32)$ $(1, 0)$

max $(-3, 32)$ Min $(1, 0)$

(c) Hence, sketch the graph of the function f on the axes below.



Done

Question 6 **(25 marks)**

The general term of an arithmetic sequence is $T_n = 15 - 2n$, where $n \in \mathbb{N}$.

(a) **(i)** Write down the first three terms of the sequence.

$T_1 = 15 - 2(1) = 13$ $\therefore$ $13, 11, 9$

$T_2 = 15 - 2(2) = 11$

$T_3 = 15 - 2(3) = 9$

(ii) Find the first negative term of the sequence.

$T_1\ T_2\ T_3\ T_4\ T_5\ T_6\ T_7\ T_8$

$13, 11, 9, 7, 5, 3, 1\ -1$

T_8 8th term.

(b) **(i)** Find $S_n = T_1 + T_2 + \cdots + T_n$, the sum of the first n terms of the series, in terms of n.

$a = 13 \quad d = -2 \quad n = n$

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[2(13) + (n-1)-2]$

$\frac{n}{2}[26 - 2n + 2] = \frac{n}{2}[28 - 2n]$

$14n - n^2$

(ii) Find the value of n for which the sum of the first n terms of the series is 0.

$S_n = 14n - n^2$

$0 = 14n - n^2$

$n^2 - 14n = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Could use -b formula $a = 1$ $b = -14$ $c = 0$

or factorize $n(n - 14) = 0$

$n = 0$ / $n - 14 = 0$

$n = 14$ → $n = 14$

Section B — Contexts and Applications — 150 marks

Answer **all three** questions from this section.

Question 7 **(35 marks)**

(a) Mary bought a new car for €20 000 on the 1st July 2010.
The value of the car depreciated at a compound rate of 15% each year.
Find the value of the car, correct to the nearest euro, on the 1st July 2014.

2010 – 2011 – 2012 – 2013 – 2014 (1, 2, 3, 4)

4years.

$\therefore A = P(1-i)^n = 20000(1-\cdot 15)^4$

$20000(\cdot 85)^4$

$(20000)(\cdot 52200) = €10\,440$

(b) Mary wishes to buy a new car, which costs €24 000, on the 1st July 2014.

(i) *Buy Right Car Sales* offers Mary €10 500 for her old car. She can borrow the balance for one year at a rate of 11·5%. How much would she repay on 1st July 2015?

$24000 - 10{,}500 = 13{,}500$ needs

$\frac{13500}{100} \times \frac{11\cdot 5}{1} = €1552\cdot 5$ interest

$\therefore$ 13500 + 1552·5 = 15052·5

$\therefore$ €15052·5

(ii) *Bargain Deals Car Sales* offers Mary €10 000 for her old car and an interest free loan of the balance for six months. At the end of the six months Mary would make a payment of €4000 and would be charged interest at a compound rate of 1·5% per month for the next six months. How much would Mary repay on 1st July 2015?

① 24000 − 10000 = 14,000 needed

after 6 months ② 14000 − 4000 = 10,000 due. $\frac{1{\cdot}5}{100} = {\cdot}015$

③ $A = P(1+r)^n$ n = no of months = 6

r = rate % = $\frac{r\%}{100} = {\cdot}015$

P = principle = 10000

$A = 10000(1+{\cdot}015)^6$

$= 10000(1{\cdot}015)^6$

$10000(1{\cdot}09344)$

$= 10934{\cdot}4$

(iii) Which of the above options should Mary choose if she wishes to pay the least amount? Justify your answer by calculation.

Option 1 6(i)

Pays 15052·5 real money.

Option 2

10934·4
+ 4000·00

14934·6 real money

∴ ✓ this one

Question 8 **(40 marks)**

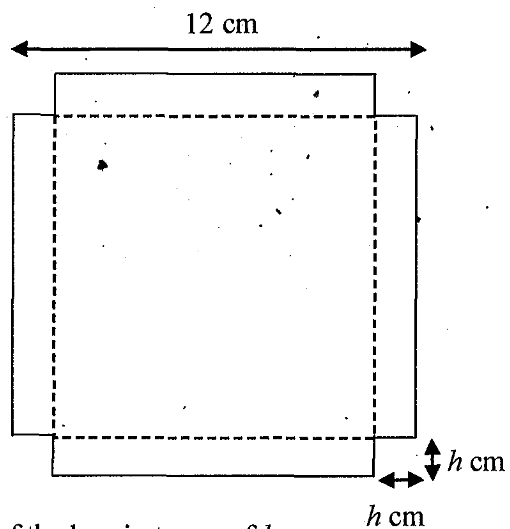
(a) The length of the side of a square sheet of cardboard is 12 cm. Find the area of the sheet.

12

12

144 cm^2

(b) The diagram below shows a square sheet of cardboard of side length 12 cm, from which four small squares, each of side length h, have been removed. The sheet can be folded to form an open rectangular box of height h.



Write the length and the width of the box in terms of h.

Length of box = $12 - 2h$

Width of box = $12 - 2h$.

(c) Show that the volume of the box, in terms of h, is $4h^3 - 48h^2 + 144h$.

$$\overset{L}{(12-2h)}\ \overset{B}{(12-2h)}\ \overset{h}{(h)}.$$

$$= (12-2h)(12h-2h^2)$$

$$12(12h-2h^2)-2h(12h-2h^2)$$

$$144h - 24h^2 - 24h^2 + 4h^3$$

$$= 4h^3 - 48h^2 + 144h$$

(d) Find the value of h which gives the maximum volume of the box.

$V = 4h^3 - 48h^2 + 144h$

$\frac{dV}{dh} = 12h^2 - 96h + 144$

$0 = 12h^2 - 96h + 144$

$\div 12 \quad 0 = h^2 - 8h + 12$

$h - 6$

$h - 2$

$h - 6 = 0 \;/\; h - 2 = 0$

$h = 6 \;/\; h = 2$

$h = 2$

nB

use -b formula

$a = 12$

$b = -96$

$c = +144$

cant be 6 as

$2h = 12$

(e) Find the maximum volume of the box.

$V = 4h^3 - 48h^2 + 144h$

$h = 2$

$V = 4(2)^3 - 48(2)^2 + 144(2)$

$4(8) - 48(4) + 288$

$32 - 192 + 288$

$288 + 32 - 192$

$320 - 192 = 128 \text{ cm}^3$

Question 9 (75 marks)

A small rocket is fired into the air from a fixed position on the ground. Its flight lasts ten seconds. The height, in metres, of the rocket above the ground after t seconds is given by

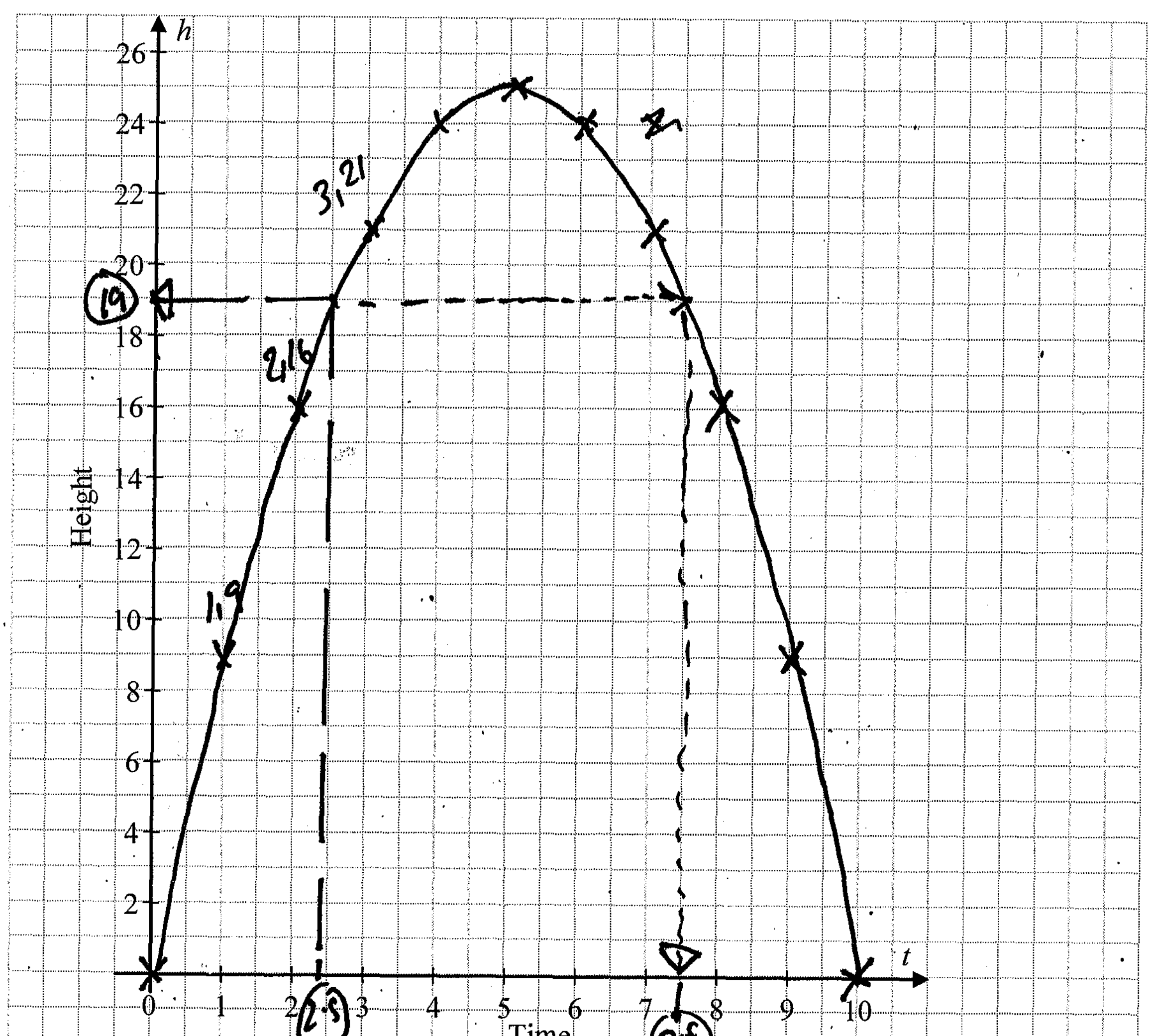
$$h = 10t - t^2.$$

(a) Complete the table below.

Time, t	0	1	2	3	4	5	6	7	8	9	10
Height, h	0	9	16	21	24	25	24	21	16	9	0

t	0	1	2	3	4	5	6	7	8	9	10
$10t$	0	10	20	30	40	50	60	70	80	90	100
$-t^2$	0	-1	-4	-9	-16	-25	-36	-49	-64	-81	-100
h	0	9	16	21	24	25	24	21	16	9	0

(b) Draw a graph to represent the height of the rocket during the ten seconds.



(c) Use your graph to estimate:

(i) The height of the rocket after 2·5 seconds. 19m

(ii) The time when the rocket will again be at this height. 7·5 secs

(iii) The co-ordinates of the highest point reached by the rocket. (5, 25)

(d) **(i)** Find the slope of the line joining the points (6, 24) and (7, 21).

$x_1 y_1 \quad x_2 y_2$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{21-24}{7-6} = \frac{-3}{1} = -3$$

(ii) Would you expect the line joining the points (7, 21) and (8, 16) to be steeper than the line joining (6, 24) and (7, 21) or not? Give a reason for your answer.

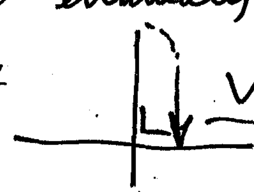
7, 21   8, 16

$x_1 y_1 \quad x_2 y_2$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{16-21}{8-7} = \frac{-5}{1} = -5$$

Answer yes because it is as calculated

but as the rocket falls from slope 0 at the max the −1, −2, −3, −4, −5, etc as it fall if it was high enough — it would eventually fall with infinite slope



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(e) **(i)** Find $\frac{dh}{dt}$.

$h = 10t - t^2$

$dh/dt = 10 - 2t$

(ii) Hence, find the maximum height reached by the rocket.

$10 - 2t = 0$

$10 = 2t$

$t = 5$

$\therefore$ sub $t = 5$ into

$h = 10t - t^2$

$h = 10(5) - (5)^2 = 25$

$\therefore$ $(5, 25)$ max

(iii) Find the speed of the rocket after 3 seconds.

dh/dt = speed $= 10 - 2t$.

at $t = 3$

$10 - 2(3) = 10 - 6 = 4$ m/sec.

(f) Find the co-ordinates of the point at which the slope of the tangent to the graph is 2.

(i) $dh/dt = 10 - 2t$.

Slope $= 10 - 2t$

$2 = 10 - 2t$.

$2t = 10 - 2$

$2t = 8$

$t = 4$ when $t = 4$

(ii) Coordinate

sub $t = 4$ into original

$h = 10t - t^2$

$h = 10(4) - 4^2$

$40 - 16 = 24$

$\therefore$ $(4, 24)$